

nag_kelvin_bei (s19abc)

1. Purpose

nag_kelvin_bei (s19abc) returns a value for the Kelvin function $\text{bei } x$.

2. Specification

```
#include <nag.h>
#include <nags.h>
```

```
double nag_kelvin_bei(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the Kelvin function $\text{bei } x$.

The function is based on several Chebyshev expansions.

For large x , there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the function must fail.

4. Parameters

x

Input: the argument x of the function.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT

On entry, $|x|$ must not be greater than $\langle value \rangle$: $x = \langle value \rangle$.

$|x|$ is too large for an accurate result to be returned and the function returns zero.

6. Further Comments

6.1. Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let E be the absolute error in the function, and δ be the relative error in the argument. If δ is somewhat larger than the **machine precision**, then we have $E \simeq |x(-\text{ber}_1 x + \text{bei}_1 x)/\sqrt{2}| \delta$ (provided E is within machine bounds).

For small x the error amplification is insignificant and thus the absolute error is effectively bounded by the **machine precision**.

For medium and large x , the error behaviour is oscillatory and its amplitude grows like $\sqrt{x/2\pi}e^{x/\sqrt{2}}$. Therefore it is impossible to calculate the functions with any accuracy when $\sqrt{x}e^{x/\sqrt{2}} > \sqrt{2\pi}/\delta$. Note that this value of x is much smaller than the minimum value of x for which the function overflows.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9.9 p 379.

7. See Also

nag_kelvin_ber (s19aac)
nag_kelvin_ker (s19acc)
nag_kelvin_kei (s19adc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_kelvin_bei(s19abc) Example Program
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
    double x, y;

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("s19abc Example Program Results\\n\\n");
    Vprintf("      x      y\\n\\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s19abc(x, NAGERR_DEFAULT);
        Vprintf("%12.3e%12.3e\\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s19abc Example Program Data
      0.1
      1.0
      2.5
      5.0
     10.0
     15.0
     -1.0
```

8.3. Program Results

```
s19abc Example Program Results
```

x	y
1.000e-01	2.500e-03
1.000e+00	2.496e-01
2.500e+00	1.457e+00
5.000e+00	1.160e-01
1.000e+01	5.637e+01
1.500e+01	-2.953e+03
-1.000e+00	2.496e-01